

The Pattern of Empty Bands

Mark Dettinger, a student at the University of Ulm (Germany), explains the pattern of empty bands (in the [table of all solution counts](#) for [Problem 3 on Day 1 of IOI'94](#)) as follows.

First, notice that the digit sum of a five-digit prime is never a multiple of three (because otherwise it would be divisible by three).

Second, observe that the prime in the rightmost column consists of digits that are each the least significant digit of a (horizontal) prime. Hence, all these digits are odd (because otherwise the horizontal prime would be even) and, thus, the digit sum is odd as well.

Consequently, the digit sum is odd and not a multiple of three. This leaves only numbers that are either 1 or 5 modulo 6. This explains the empty bands in the [table of all solution counts](#): 2, 8, 14, 20, 26, 32, 38, and 44 are 2 modulo 6; 4, 10, 16, 22, 28, 34, and 40 are 4 modulo 6.

This still does not explain why there are no solutions with digit sum 5, 7, 41, or 43. Of course, there are only a few primes with these digit sums (12, 28, 13, and 4 respectively), so it is more difficult to make things fit.